Projective geometry, like Euclidean geometry, can be developed both from a synthetic (axiomatic) and analytic point of view. In the analytic approach, a projective space of dimension \( n \) is now a vector space of dimension \( n + 1 \); we then get not only points and lines but higher-dimensional analogues of planes. (At least, this is the way I learned the definition; as we'll shortly see, the definition in this book is somewhat different.) Projective geometry is an extension (or a simplification, depending on point of view) of Euclidean geometry, in which there is no concept of distance or angle measure. Intuitively, projective geometry can be understood as only having points and lines; in other words, while Euclidean geometry can be informally viewed as the study of straightedge and compass constructions, projective geometry can be viewed as the study of straightedge only constructions. Surprisingly, though originally conceived as a simplification, projective geometry is not just a subset of Euclidean geometry. It may seem similar since it seems to deal primarily with the projection of Euclidean objects on Euclidean planes. But that is not all it does. Think about our example of the pair of railroad tracks converging on the horizon. In your painting of the tracks, the two lines representing them meet at a point on your canvas, but what does that point represent in the real world? Projective Geometry. Milivoje Lukić. Abstract. Perspectivity is the projection of objects from a point. A quantity that is preserved by this map, called the cross-ratio, naturally appears in many geometrical configurations. This map and its properties are very useful in a variety of geometry problems. We study the pole of a line, polar of a point, cross ratio, perspectivity, and we prove theorems of Desargue, Pappus, Pascal, Brianchon, and Brokard. Projective geometry is an elementary non-metrical form of geometry, meaning that it is not based on a concept of distance. In two dimensions it begins with the study of configurations of points and lines. This early 19th century projective geometry was intermediate from analytic geometry to algebraic geometry.

Projective geometry is concerned with the properties of figures that are invariant by projecting and taking sections. It is considered one of the most beautiful parts of geometry and plays a central role because its specializations cover the whole of the affine, Euclidean and non-Euclidean geometries. The natural extension of projective geometry is projective algebraic geometry, a rich and active field of research. Regarding its applications, results and techniques of projective geometry are today intensively used in computer vision. This book contains a comprehensive presentation of projective geometry, over the real and complex number fields, and its applications to affine and Euclidean geometries. It covers central topics such as linear varieties, cross ratio, duality, projective transformations, quadrics and their classifications – projective, affine and metric –, as well as the more advanced and less usual spaces of quadrics, rational normal curves, line complexes and the classifications of collineations, pencils of quadrics and correlations. Two appendices are devoted to the projective foundations of perspective and to the projective models of plane non-Euclidean geometries. The presentation uses modern language, is based on linear algebra and provides complete proofs. Exercises are proposed at the end of each chapter; many of them are beautiful classical results.

The material in this book is suitable for courses on projective geometry for undergraduate students, with a working knowledge of a standard first course on linear algebra. The text is a valuable guide to graduate students and researchers working in areas using or related to projective geometry, such as algebraic geometry and computer vision, and to anyone wishing to gain an advanced view on geometry as a whole.

Keywords: Projective geometry, affine geometry, Euclidean geometry, linear varieties, cross ratio, projectivities, quadrics, pencils of quadrics, correlations