Noncommutative geometry of homogenized quantum $\mathfrak{sl}(2, \mathbb{C})$

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Abstract

We examine the relationship between certain noncommutative analogues of projective 3-space, $\mathbb{P}^3$, and the quantized enveloping algebras $U_q(\mathfrak{sl}_2)$. The relationship is mediated by certain noncommutative graded algebras $S$, one for each $q \in \mathbb{C}^\times$, having a degree-two central element $c$ such that $S[c^{-1}]_0 \cong U_q(\mathfrak{sl}_2)$. The noncommutative analogues of $\mathbb{P}^3$ are the spaces $\text{Proj}_{\text{nc}}(S)$. We show how the points, fat points, lines, and quadrics, in $\text{Proj}_{\text{nc}}(S)$, and their incidence relations, correspond to finite-dimensional irreducible representations of $U_q(\mathfrak{sl}_2)$, Verma modules, annihilators of Verma modules, and homomorphisms between them.

Keywords

noncommutative algebraic geometry, quantum groups, quantum $\mathfrak{sl}_2$

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Noncommutative Geometry and quantum statistical mechanical systems. by Matthew Terje Aadne. This innerproduct turns $L^2(X, \mu)$ into a Hilbert space. In the chapters that follow we will be especially interested in the case where the measure space $(X, A, \mu)$ is discrete, meaning that the sigma algebra $A$ is the power set and $\mu$ is the counting measure. In this case we use the notation $l^2(X, \mu)$ instead of $L^2(X, \mu)$. In the discrete case we have a particularly handy basis for $l^2(X, \mu)$. For each $x \in X$ let $x : X \to \mathbb{C}$ be defined by. $1$ if $y = x$ $x(y) = 0$ otherwise. The collection forms an orthonormal basis for $l^2(X, \mu)$, meaning that then if $S \cap S^{-1} = \{e\}$ we may define a partial order $\preceq_S$ on $G(S)$, turning it into a partially ordered group, by. "The Noncommutative Algebraic Geometry of Quantum Projective Spaces," a dissertation prepared by Peter D. Goetz in partial fulfillment of the requirements for the Doctor of Philosophy degree in the Department of Mathematics. This dissertation has been approved and accepted by. AREAS OF SPECIAL INTEREST: Noncommutative Ring Theory Algebraic Geometry. PROFESSIONAL EXPERIENCE: Graduate Teaching Assistant, Department of Mathematics, University of Oregon, Eugene, 1999-2003. In Construction 1 we build an Ore extension of the rst homogenized Weyl algebra $A_1(k)$. From this we construct a one parameter family of generic quantum $\mathbb{P}^3$'s, denoted by $S_{\alpha}$, $\alpha \in k^*$. We refer the reader to III.2 for the definition of $S_{\alpha}$. The main result is Theorem III.2.7 The softening of a quantum group is considered, and we introduce $q$ curvatures satisfying $q$ Bianchi identities, a basic ingredient for the construction of $q$ gravity and $q$ gauge theories. An introduction to noncommutative differential geometry on quantum groups. @article{Aschieri1993ANIT, title={AN INTRODUCTION TO NONCOMMUTATIVE DIFFERENTIAL GEOMETRY ON QUANTUM GROUPS}, author={Paolo Aschieri and Leonardo Castellani}, journal={International Journal of Modern Physics A}, year={1993}, volume={08}, pages={1667-1706} }. Paolo Aschieri, Leonardo Castellani. Published 1993. Noncommutative geometry (NCG) is a branch of mathematics concerned with a geometric approach to noncommutative algebras, and with the construction of spaces that are locally presented by noncommutative algebras of functions (possibly in some generalized sense). A noncommutative algebra is an associative algebra in which the multiplication is not commutative, that is, for which. does not always equal. Noncommutative geometry. Quite the same Wikipedia. Just better.