Book Reviews


I. INTRODUCTION

The book under review is concerned with jump linear systems in discrete time, where the jumps occur at random times following a discrete-time Markov chain. It mainly collects the authors’ work on jump linear systems in recent years. The book is written for researchers and graduate students although it is not intended to be a textbook.

Frequently, in various real-world applications, not only are the underlying dynamic systems time varying, but also they are associated with discrete movements that are influenced by uncertain and exogenous discrete events driven by random disturbances. Many of such systems involve stochastic processes of pure jump type, especially for those arising in production planning, economics, and stochastic networks. As a result, the parameters of the dynamic systems come from one of a number of different regimes with transitions among regimes governed by a jump process. The main motivation stems from taking discrete event interventions into consideration for a wide range of applications in stochastic networks, communication systems, production planning, and manufacturing. To model such systems, Markovian formulations have been found to be useful. To better reflect reality and to produce mathematically tractable models, jump linear systems are used frequently; see [1], [4]–[6], among others. For example, consider a nation’s economy, in which the discrete events are coined by economists as discrete shifts. The evolution of the economy often displays dramatic moves (ups or downs), which then is naturally modeled by a Markov chain; see [2], [3]. Suppose that one considers a linear model. Very often, the coefficients of the linear systems are not fixed but rather subject to discrete shifts in regime such that across different regimes, the behavior of the corresponding dynamic systems are markedly different. As a result, a promising alternative than the traditional model is to allow for the possibility of sudden, discrete changes in the values of the parameters resulting in a so-called jump linear system. Such a system is also referred to as a hybrid model or a regime-switching system. Using such a premise, in lieu of considering fixed matrix coefficients in the model, one may treat the case that the coefficients of the system are time varying and depend on a Markov chain. An important problem is: When the state space of the finite-state Markov chain is large, how can we reduce the computational complexity. Clearly, not all parts or components of a large-scale system evolve at the same rate. Some of them change rapidly and others vary slowly. The different rates of variations allow us to reduce complexity via decomposition and aggregation. A systematic approach is to use two-time-scale formulation to bring out the hierarchical structure of the underlying systems; see [5] and the references therein.

In this book, the authors consider systems of the following form:

\[
\begin{align*}
    x(k+1) &= A_{0(k)}x(k) + B_{0(k)}u(k) + G_{0(k)}w(k) \\
    y(k) &= L_{0(k)}x(k) + H_{0(k)}w(k) \\
    z(k) &= C_{0(k)}x(k) + D_{0(k)}u(k) \\
    x(0) &= x_0, \quad \theta(0) = \theta_0.
\end{align*}
\]

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In the above, \( x(k) \) represents the state variable, \( u(k) \) the control, \( w(k) \) the disturbance, \( y(k) \) the measurement, and \( z(k) \) the output of the system. The \( \theta(k) \) is a discrete-time Markov chain with a finite state space. As can be seen, the coefficients of the system are time varying and depend on the outcome of the Markov chain \( \theta(k) \). The authors adopt the point that the state variable is \( x(k) \) only. An alternative point of view is that the state consists of two components, namely \( x(k) \) and the Markov state \( \theta(k) \). Such a point may be beneficial when one aims to Markovianize a system.

One of the techniques used in the book is the linear matrix inequality (LMI). An LMI is any constraint that can be written as or converted to

\[
    F(x) = F_0 + x_1F_1 + \cdots + x_mF_m < 0
\]

where \( x_i \) are the variables, \( F_i \in \mathbb{R}^{n_i} \) for \( i = 1, \ldots, m \), are known Hermitian matrices, and \( \mathbb{R}^{n_i} \) is the normed linear space of \( n \times n \) matrices.

In studying control and optimization of the regime-switching systems, Riccati equations of systems naturally arise. To some extent, the book under review is largely devoted to the study of coupled Riccati systems of the form

\[
    \begin{align*}
    X_i(k) &= A_i(k)^*E_i(X(k + 1, k), k)A_i(k) \\
            &\quad - A_i(k)^*E_i(X(k + 1, k), k)B_i(k) \\
            &\quad \cdot (D_i(k)^*D_i(k) + B_i(k)^*E_i(X(k + 1, k), k)B_i(k))^{-1} \\
            &\quad \cdot B_i(k)^*E_i(X(k + 1, k), k)A_i(k) + C_i(k)^*C_i(k), \\
    X_i(T) &= V_i,
    \end{align*}
\]

where the symbol \( * \) denotes conjugate transpose

\[
    E_i(V, k) = \sum_{j=1}^{N} p_{ij}(k)V_j
\]

and \( P(k) = (p_{ij}(k)) \) is the transition probability matrix. The coupling is due to the Markovian structure and is through the transition probability matrix. Much of the study throughout the book focuses on various properties of the Riccati systems. Thus, a subtitle of the book might be properties of coupled Riccati systems.

II. ORGANIZATION OF THE BOOK

Consisting of eight chapters and three appendices, this book may be divided into three parts. The first part is an overview of the subject matters together with background materials. Part 2 is the main body of the book dealing with control, optimization, design, and stability of jump linear systems. Part 3 provides several appendices for references. Chapter 1 begins with the introduction of jump linear systems. After some introductory remarks, the authors provide a chapter by chapter overview of the main body of the book. It is emphasized that the view taken by the authors is the so-called operator theoretic approach, which should be distinguished from the multiple model approach, and the hidden Markov model approach.

Chapter 2 provides some background materials. After recalling some notations, the effort is directed to reviewing linear systems theory, including Lyapunov stability, controllability and observability, Riccati equations, and linear matrix inequalities.
Chapter 3 is on stability. Specific attention is paid to mean square stability (MSS). The discussion is divided to MSS for homogeneous systems and to nonhomogeneous systems, and stabilizability and detectability. Finally, pathwise (probability one) stability is also presented.

Chapter 4 deals with optimal controls. It contains finite horizon quadratic optimal control problems, infinite horizon quadratic optimal control problems, $H_2$ control problems, and quadratic controls with stochastic $\ell_2$ inputs.

Chapter 5 concentrates on filtering. Consider

\[
x(k+1) = A_0(k)x(k) + B_0(k)u(k) + C_0(k)w(k),
\]

\[
y(k) = L_0(k)x(k) + H_0(k)w(k)
\]

where $A_0(k)$, $B_0(k)$, $C_0(k)$, $L_0(k)$, and $H_0(k)$ are system coefficients with appropriate dimensions, $w(k)$ is the noise, and $\theta(k)$ is a discrete-time Markov chain. The filtering problems are divided according to if finite or infinite horizon is used and/or if the $\theta(k)$ is known. Optimal filters with unknown $\theta(k)$ is considered together with robust linear filters via LMI (linear matrix inequality) formulation.

While Chapter 4 deals with completely observable case, Chapter 6 focuses on quadratic optimal controls with partial information. Naturally, one needs to combine control methodology with filtering techniques. Here again, finite horizon and infinite horizon problems are treated. Separation principle is studied along with $H_2$ controls.

Chapter 7 is concerned with worst case scenario. The setup here is on the $H_\infty$ controls. It is assumed that the controller has access to both the state variable and the jump variables. The problem is setup in a certain Hilbert space. Necessary and sufficient conditions for stabilizability are provided by dealing with coupled systems of algebraic Riccati equations. Recursive algorithms for the $H_\infty$ control are also provided.

Chapter 8 presents design techniques together with examples. After some applications are mentioned, it concentrates on such topics as robust control using LMI approximations, $H_\infty$ designs, linear filtering with unknown $\theta(k)$.

The three appendices provide brief accounts on coupled algebraic Riccati equations, linear filtering with unknown $\theta(k)$, and $H_2$ control problems. At the end of each chapter (except Chapter 2), a section on historical remarks is given, which provides further references among others. Finally, a tabulated notation and abbreviation is provided at the end of the book.

### III. Further Remarks

The prerequisites for reading the book are modest including some knowledge in analysis and probabilities. As the authors mentioned in the preface, they intend to put together recent results collectively, and to further stimulate subsequent research. It may be used as a reference book or in a special topic course for graduate students. The book is well written. It is a good quality book with worthy contributions to the literature and to the control and optimization community.

### REFERENCES

Prior books in the series centered a lot around club interaction, Two floundering souls collide and pure magic ignites! With the fourth book in her addictive Demon Squad MC series, Monique Moreau delights in a totally different way in STANTON'S SINS! The random review ltd said: “It is a one actual task to switch over to a different genre, let that be writing or reading. I have read couple of KEVIN MISSAL books, and there was no second thoughts before choosing this one. Find book reviews, essays, best-seller lists and news from The New York Times Book Review.”

Robert Gottlieb on Dickensworld â€” the Great Novelistâ€™s Grand Universe. On the 150th anniversary of Charles Dickensâ€™s death, Robert Gottlieb considers a new book, â€œThe Mystery of Charles Dickens,â€ by A.N. Wilson, and delivers his own assessment of the authorâ€™s legacy. November 6, 2020 By Robert Gottlieb. Nonfiction. What Is a Book Review? Traditionally, book reviews are written evaluations of a recently published book in any genre. Usually around the 500 to 700 word mark, they offer a brief description of a textâ€™s main elements while appraising the workâ€™s overall strengths and weaknesses. Published book reviews can appear in newspapers, magazines, and academic journals. They provide the reader with an overview of the book itself and indicate whether or not the reviewer would recommend the book to the reader.