

MATHEMATICS TEXTBOOK ANALYSIS: THE SIGNIFICANCE OF TEXTBOOK FEATURES TO STUDENT LEARNING

Lisa O’Keeffe and John O’Donoghue

NCE-MSTL, University of Limerick

Much international research has been carried out on the influence of mathematical textbooks with Irish textbooks playing only a minor role in the TIMSS analysis (Valverde et al, 2002). Irish students have performed poorly in the PISA reports from 2001, 2003 and again in 2006, and are currently ranked in the middle of the OECD countries for mathematical literacy (Cosgrove et al., 2005). In 2005 the NCCA identified that mathematics in Irish post primary schools is currently being taught through a didactic style of teaching with the textbook playing a central role. Since the mathematics textbook has such a pivotal role within the classroom the impact of the mathematics textbook on student learning is undeniable. This research, by means of mathematics textbook analysis, investigates the effectiveness of the current Irish Junior Cycle mathematics textbooks for developing student comprehension and motivation.

INTRODUCTION

Despite a minor inclusion in the TIMSS Report no Irish research has been carried out to evaluate the effectiveness of the textbooks currently in use (Valverde et al, 2002). This lack of research from an Irish perspective justifies an analysis of the current mathematical textbooks employed at second level in Ireland. In a minor study conducted by O’Keeffe (2007) it was noted that over 75% of Irish secondary school teachers surveyed use only one mathematics textbook on a daily basis. The National Council for Curriculum and Assessment (NCCA) consulted stakeholders, e.g. teachers, parents and students nationally and put forward a number of recommendations, one of these being to improve the textbooks and available resources. Over 90% of those consulted felt strongly about the effective role an improved textbook could play in pupil learning (NCCA, 2006).

The authors conducted a mathematics textbook analysis study with the explicit intention of highlighting key textbook features which impact positively on students’ learning. They were dissatisfied that a blanket evaluation of the textbooks could be conducted without including a language analysis element. Therefore a framework for textbook analysis that integrates textbook language analysis and other elements based on the work of Halliday (1973), Morgan (2004), the TIMSS study (Valverde et al., (2002)) and Rivers (1990) was developed. This framework comprises of four key elements: Content, Structure, Expectation and Language. The purpose of adding the language element is to strengthen their textbook analysis framework allowing for effective analysis of the textbook as a whole. Language analysis and its significance to teaching and learning has been widely researched for a number of years and has featured significantly in mathematics education research since the early 1990’s, (e.g.

in the work of Halliday, 1973; Skemp, 1982; Pimm, 1987; Noonan, 1990; Chapman, 1993; Mikk, 2000; Morgan, 2004; Orton, 2004 and Herbel-Eisemann & Wagner, 2005). However, a suitable method for mathematics language analysis has yet to be applied to mathematics textbook analysis. The authors are attempting to bridge this gap as part of their research by drawing on the work of Halliday (1973), Morgan (2004) and, to a lesser extent, Herbel-Eisemann & Wagner (2005).

Textbook research in Ireland is timely in the context of the national curriculum change which is currently underway at secondary level and is referred to as Project Maths. New classroom materials and textbooks will be required to support this new curriculum initiative, thus providing an opportunity for effective textbooks to be created. This paper provides an in-depth look at key textbook design features, emerging from the textbook analysis, which impact on students' comprehension and motivation.

METHODOLOGY

The mathematics textbook analysis is carried out using a combination of researched frameworks for textual analysis based primarily on the TIMSS analysis. Table 1 provides an outline of the theoretical frameworks which were considered and applied in this research study.

Table 1: Theoretical Frameworks, their Influence and Significance

Theoretical Framework	Influence	Significance
TIMSS, 2002	Provides structure for textbook analysis	* Content Analysis * Structure Analysis * Expectation Analysis
Rivers, 1990	Reinforce the TIMSS framework	* Content Analysis * Expectation Analysis
Mikk, 2000	Reinforce the TIMSS framework	* Structure Analysis
Morgan, 2004	Provides framework for language analysis	* Language Analysis

The textbook analysis employed in this study comprises four key elements; Content, Structure, Expectation and Language and is applied to the three most commonly used mathematics textbook series at lower secondary school in Ireland. The content of each textbook was analysed using TIMSS content analysis combined with the 'River's Matrix', (Rivers, 1990). The structure of each textbook is analysed using a combination of the TIMSS framework which focuses on the physical scale of a textbook and a method for analysis devised by Mikk (2000), whereby the structure of a text is recorded diagrammatically. The structure of each textbook was analysed by employing both of these frameworks and a table was created for comparison using

a number of headings such as Block Type, Content Structure and Expectation. Following this, the Physical Structure of each textbook is also examined. Performance expectations are embedded throughout the textbooks and these can impact significantly on how pupils choose to deal with the topics presented. The most basic consideration of expectation analysis is that pupils and teachers alike will read and understand the material presented (Valverde et al., 2002). The textbook expectations were analysed using a combination of both the TIMSS expectation analysis (Valverde et al., 2002) and the 'River's Matrix', (Rivers, 1990).

The language analysis draws primarily upon the work of Halliday (1973) and Morgan (2004). Halliday's research provides the basis for much language analysis in many different subject areas focusing on the functional aspects of language. He outlines this functional aspect as the way in which language is used, the purpose that it serves and the way in which a reader can achieve these purposes. Halliday's functional grammar analysis is based on three elements: Ideational Function, Interpersonal Function and Textual Function.

The Ideational Function looks at the nature of the activity, in particular at the structure and logic of relationships with a text (any written material). Halliday, (1973:38) describes it as "the categories of one's experience of the world and how they interpret this experience". In this analysis the ideational function was analysed using the following headings (as provided by Morgan (2004))

Table 2: The Ideational Function

	Human		Object			Notation
	General	Specific	Basic or Derived	Relational	Representational	Passive Sentences
Material						
Mental						
Relational						
Verbal						
Behavioural						
Existential						

The first column indicates the six main types of processes identified by Halliday (1973), which indicate the various types of mathematical text, i.e. the extent of the presence within the text and the sort of processes they are activities present in any text. Morgan (1995) indicates that the frequency of each of the types of processes present in a text indicates the nature of the mathematical activity. Each sentence within the selected text was categorised into one of the Halliday's six process categories and the number sentences of each process (material, mental etc) was recorded.

The Interpersonal Function examines the social and personal relationships between the textbook author and others while establishing the expression of the author's authority and the relationship between the author and reader. Halliday, (1973:41) defines his interpersonal function as "including all forms of the speaker's intrusion into the speech situation and speech act". As highlighted by Morgan (2004:84) the obvious measure of the interpersonal function is through the use of personal pronouns and imperatives. Morgan (1995) states that imperatives, which are

directly associated with the reader, support a claim from the author that he/she is a member of the mathematical community. Such a mathematical community uses specialist vocabulary which the student may not be familiar with, thus allowing the textbook author to project an authoritative voice. In this phase of language analysis a count of the various pronouns, symbols, imperatives and informal sentences was conducted. For the purpose of this study an informal sentence is interpreted as any chat-like sentence within the textbook.

The Textual Function identifies whether the language used is relevant for its intended purpose. Halliday (1973:42) points out that it “distinguishes a living message from a mere entry in a grammar or a dictionary”. Identifying the overall theme put forward by a text can only be considered in conjunction with the ideational and interpersonal aspects. By constructing a thematic overview of a text one can then conclude on the nature of the discourse and its relevance to its intended purpose. Morgan (1995) suggests that due to the reasoning/deductive nature of mathematics one would expect to uncover a focus on logical reasoning and the progression of argument. This study analysed the textual function by examining the presence of reasoning conjunctions (e.g. as a result of, even though), logical reasoning conjunctions (e.g. basis for, thus) and the use of reasoning imperatives (e.g. Next, Then).

Morgan's framework (2004) for mathematics language analysis which originated from the work of Halliday (1973) originated can be applied effectively to any mathematical text. While Halliday's framework for language analysis was not mathematics specific it was adapted by Morgan for analysing student's own mathematical writings. In this research study Morgan's framework for language analysis is applied to school mathematics textbooks.

Using a combination of the four key elements of the framework for textbook analysis, (Content, Structure, Expectation and Language) this paper focuses on the impact of the textbooks on a students' learning and in particular on their comprehension and motivation. The three most commonly used Irish Junior Cycle (lower secondary school) mathematics textbook series are used in this study. Each textbook series contains two textbooks and was randomly labelled TBS A, TBS B and TBS C and within each textbook series the first textbook was numbered 1 and the second textbook was numbered 2. For example: TBS A1 is Textbook Series A – Book 1. For the purpose of language analysis the first page of each chapter from each textbook was selected as this page is consistently language rich.

MAIN FINDINGS

According to Mikk (2000) new information motivates students, however, he states that only 30-40% of the information presented to students should be new information. The reality is students encounter 70 – 80% new information in their mathematics textbooks. To counteract this Rivers and Mikk suggest that a number of factors such as historical data, practical implications, humour, figurative

representation and narration can serve to enhance students' motivation. The findings from the Rivers Matrix outline the absence of such motivational factors in the mathematics textbooks involved in this study. The presence of historical notes, biographies, career information and photographs is almost non-existent across most of the textbooks. The percentage of exercises which are problems is particularly low in each case. Given that Pisa (2006) highlights how only 10% of Irish students fall into the higher brackets with regard to mathematical knowledge (the OECD average is 13%), the lack of emphasis on problem solving is not surprising.

Noonan (1990: 60) states that “comprehension of mathematics texts for children is improved when there are illustrations”. This is reinforced by Mikk (2000) and Rivers (1990). Illustrations can grab and hold a students attention. From the textbook analysis it is evident that the use of attractive colours is limited throughout all textbooks and there is a lack of consistency across colour use. For all textbooks the colour schemes tend to depend on a grouping of chapters as opposed to common topics and themes throughout. Yellow is the most common colour evident throughout all textbook series; however the shade of yellow which is used in the mathematics textbooks is extremely dull and faded.

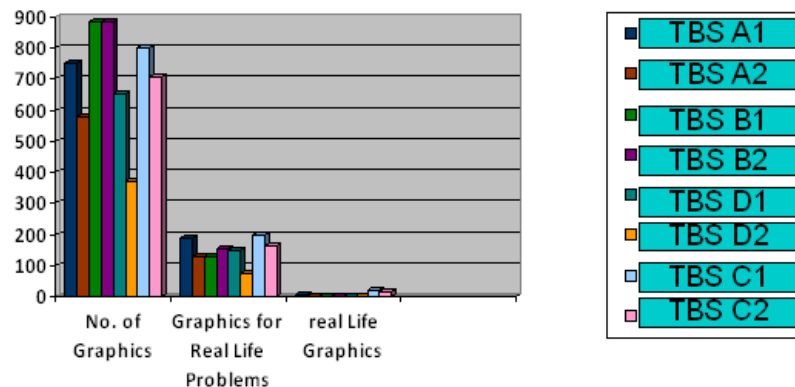


Figure 1: Distribution of Graphics throughout the Junior Cycle Mathematics Textbooks

There is a multitude of graphics present in all textbooks (Figure 1), however the number of graphics assisting real life problems (20.6%) or indeed real life graphics (0.86%) is low in comparison to the number of graphics used throughout the textbooks. The majority of graphics in the textbooks are simple mathematical representations such as triangles, parallel lines angles etc.

The language analysis identified a number of interesting findings which impact directly on students learning. The following table outlines some of the findings which impact directly on student comprehension and motivation of mathematics.

Table 3: Instances of Key Language Features

Text-book:	Pronoun 'we'	*Other Pronouns	Passive sentences	Specialist words	Symbolism	Informal Sentences	Imperatives
TBS A1	63	17	68	208	117	7	37
TBS A2	23	14	17	125	216	9	31
TBS B1	17	29	73	317	197	2	42
TBS B2	21	27	44	312	309	2	58
TBS C1	21	42	34	135	115	63	6
TBS C2	9	15	20	61	23	9	6

*Other pronouns include I, You, Me, My, Their, Our, Your (all appear in at least one of the textbooks)

** TBS A1 contains 27 chapters and 441 pages, TBS A2 contains 14 chapters and 320 pages, TBS B1 contains 27 chapter and 420 pages, TBS B2 contains 28 chapters and 442 pages, TBS C1 contains 21 chapters and 438 pages and TBS C2 contains 14 chapters and 394 pages.

Pimm (1987) highlights the relationships suggested by the pronoun 'we', such relationships may be harmful for students who are not yet ready to take on any shared responsibility. 'We' is also reflective of the author's personal involvement in the text and the readers assumed interest. Passive sentences are also indicative of the nature of a textbook and overuse removes the purpose of human activity from mathematics which is counter productive to active learning (Morgan, 1995). While specialist words and symbols are necessary for the language of mathematics excessive use of these in a textbook is unnecessary for the reader. An overuse of symbols and specialist vocabulary hinders a student's comprehension of the text. Pimm (1987) also reinforces Morgan's beliefs about the use of imperatives. Morgan (1995) states that imperatives, which are directly associated with the reader, support a claim from the author that he/she is a member of the mathematical community. In his work, Rotman (2006) distinguishes between inclusive and exclusive imperatives. He suggests that inclusive imperatives ask the reader to be a thinker while exclusive imperatives ask the reader to a scribbler. Of the imperatives in the textbooks involved in this study no more than 11% of the imperatives can be considered inclusive imperatives.

DISCUSSION

According to Rivers (1990) textbooks should offer innovative resources. None of the textbooks examined were innovative in this sense. Across all three textbook series studied no reference to careers was evident. Only three textbooks have any historical references, with two of these including biographies. As regard to problem solving, less than one quarter of all exercises present in all the textbooks could be classified as problems with the highest presence of problems evident in the TBS C. In general it appears that the TBS C offers the greatest potential for pupil motivation. Colour, and the presence and correct use of diagrams play a vital role in pupil comprehension.

The use of colour for background, font and graphical demonstrations appears to be limited across all textbooks. However, it is worth remembering in this context that Dowling (1996) states that excessive and unnecessary use of colour can hinder comprehension. As outlined yellow, blue and black are the most commonly used textbook colours. However, only pale dull shades of each colour, as opposed to bright vibrant colours, are used in these textbook. Many researchers have suggested the usefulness of relevant, bright, attractive illustrations (Dowling, 1996; Mikk, 2000; Noonan, 1990; Rivers, 1990). Not only do graphics assist with pupils' understanding of the mathematical topic/problem, they also assist in grabbing and holding a pupils' attention (Mikk, 2000). While there is no shortage of graphics throughout any of the textbooks, the presence of real life graphics is extremely low with four textbooks having no real life graphics.

The key language features which impact on student's comprehension and motivation are outlined in Table 3. As outlined by Morgan (2004) any academic mathematical text will have a higher occurrence of the pronoun 'we' and the author's position is very much that he/she is an established member of the mathematical community. However, school mathematics texts, while they are designed for an academic purpose differ greatly from academic texts in terms of intention. The intention of a school mathematics text is to support teaching and learning. Morgan (2004) also talks about how this intention difference is a source of tension. According to Shuard & Rothery (1984) the main goal of a school texts are to teach concepts, skills and problem solving strategies, provide opportunity to practice, revise, and test these while developing one's mathematical language. These goals differ greatly from those of an academic text. Thus a school text should not be directly comparable to an academic text and text authors should be aware of the implications of not concerning themselves with who the intended reader will be. The high occurrence of the pronoun 'we' may be particularly negative for students who lack confidence. Fortanet (2004) speaks about how lecturers often use 'we' as a means of building up a rapport but that in fact they are rhetorical indicators. Also Kamio (2001) echoes Pimm (1987) when he highlights the close relationship that the pronoun 'we' suggests exists between the reader and the textbook author. Passive sentences remove the human participant from the activity and hence are counterproductive for an active learning environment.

Specialist vocabulary and symbolism need to be carefully planned throughout a textbook. High volumes of new words and symbols cause reading difficulties for students and are reflective of the type of expectations that the textbook author is making of the students. Unexplained or excessive use of specialist words and symbolism can create confusion and hinder a student's reading of the text while a high frequency of imperatives suggest direct orders which is reflective of procedural learning. The high rate of symbolism and specialist vocabulary also suggests that the textbook author believes that the reader is also a member of this 'mathematical community' and as such is a colleague of sorts. The low level of informal sentences present in TBS A and B supports this view of the author and reader belonging to the

mathematical community. In contrast to this, the presence of the informal sentences can relax the tone of a textbook which would be of great benefit to under-confident students.

The type of conjunctions present in a textbook reflects the nature of the textbook. In general TBS C is the only textbook series which considers the narrative, with a high emphasis on encouraging students to read. This is reflected in the choice of pronouns, presence of informal sentences and 'stories' and the lower levels of specialist vocabulary and symbolism. We expect deductive and reasoning conjunctions to be more prevalent in a mathematics textbook due to its intended role in assisting student learning. A high rate of recall and recount conjunctions (such as 'then') are reflective of procedural learning and such emphasis fails to encourage students to engage in mathematics.

Despite the fact that O'Keeffe (2007) noted that over 50% of teachers in her study rely specifically on TBS A for classroom planning and teaching, it appears from the current analysis that TBS C is to be more beneficial for student learning. As measured TBS C provides the greatest opportunity for motivation e.g. it has a greater number of graphics present and it incorporates a number of graphics in assisting real life problems and real life graphics. TBS C also suggests a greater understanding of its intended reader with the low rate of specialist vocabulary and symbolism and higher rate of informal sentences. TBS C also has a narrative theme which encourages reading, thus highlighting the relevance of reading mathematics. However, this paper is not suggesting that this textbook is the most beneficial for student learning, but rather it is the best available in the current situation.

CONCLUSION

Textbooks have a significant impact on student learning due their pivotal role in classrooms. It is expected that on-going investigations will identify some of the key textbook features while also highlighting the significance of an improved textbook for both the teaching and learning of mathematics. This is particularly important in an Irish context where almost 50% of teachers currently teaching mathematics at second level in Ireland do not have a mathematics specific qualification (Ni Riardáin & Hannigan, 2009) and the majority of qualified mathematics teachers in second level schools are teaching at Leaving Certificate level (upper secondary school). This indicates that Junior Cycle Mathematics is poorly populated with qualified Mathematics teachers, and this reality underlines the need for improved mathematics textbooks.

Many researchers have created complex and difficult to implement methods of language analysis such as the Dale-Chall list or time consuming non mathematics specific methods such as 'Cloze Tests'. The method for mathematics textbook language analysis developed by the authors can be applied to any mathematical text, does not require a large testing phase with students and can be used in conjunction

with other methods of textbook analysis. This research identifies a single framework for mathematics textbook analysis which encompasses all areas of the mathematics textbooks and by adapting the work of Morgan (2004) and Halliday (1973), this research also provides an effective measure of the language in mathematics textbooks.

REFERENCES

- Chapman, A., (1993), Language and Learning in School Mathematics: A Social Semiotic Perspective, *Issues in Educational Research*, 3(1), 35-46
- Cosgrove, J., G. Shiel, N. Sofroniou, S. Zastrutski, and F. Shortt (2005). *Education for Life: The Achievements of 15 year olds in Ireland in the Second Cycle of the PISA*. Dublin: Educational Research Centre.
- Dowling, P., (1996), A Sociological Analysis of School Mathematics Texts, *Educational Studies in Mathematics*, 31(4), 389-485
- Fortanet, I., (2004), The Use of 'we' in University Lectures: Reference and Function, *English for Specific Purposes*, 23, 45-66
- Halliday, M., (1973), *Explorations in the Functions of Language*, London: Edward Arnold
- Herbel-Eisenmann, B. and D. Wagner (2005). In the middle of nowhere: How a textbook can position the mathematics learner. In H. L. Chick and J. Vincent (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education*, Volume 3, Melbourne, pp. 121-128
- Hiebert, J. and T. Carpenter (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Education*, pp. 65-97. New York: Macmillain Publishing Company.
- Hiebert, J. and R. Gallimore and H. Garnier and K.B. Givvin and H. Hollingsworth and J. Jacobs and A. Chui and D. Wearne and M. Smith and N. Kersting and A. Manaster and E. Tseng and W. Etterbeck and C. Manaster and P. Gonzales and J. Stigler, (2003), *Teaching Mathematics in seven countries: Results from the TIMSS 1999 Video Study*, March, Washington, DC: National Centre for Education Statistics
- Kamio, A., (2001), English Generic we, you and they: An Analysis in terms of territory of information, *Journal of Pragmatics*, 33, 1111-1124
- Mikk, J., (2000), *Textbook: Research and Writing*, Oxford: Lang
- Morgan, C., (1995), *An Analysis of the Discourse of Written Reports of Investigative Work in GCSE Mathematics*, Unpublished, University of London
- Morgan, C., (2004), *Writing Mathematically: The Discourse of Investigation*, London: Falmer Press

- NCCA, (2006), Review of Mathematics in Post primary Education: Report on the Consultation, Dublin: Government Publications
- Ni Riardáin, M., & A. Hannigan, (2009), Out-of-Field Teaching in Post primary Mathematics Education, Research Report, NCE-MSTL, University of Limerick.
- Noonan, J., (1990), Readability Problems presented by Mathematics Texts, *Early Child Development and Care*, 54, 57-81
- O'Keeffe, L., (2007), An Analysis of the Junior Cycle Mathematics Textbooks, Unpublished Dissertation, University of Limerick
- Orton, A., (2004), Learning Mathematics: Issues, Theories and Classroom Practice, London: Continuum
- Pimm, D., (1987), Speaking Mathematically: Communication in Mathematics Classrooms, London: Routledge Press
- Rivers, J., (1990), Contextual Analysis of Problems in Algebra 1 Textbooks, University of South Carolina, presented at the annual meeting of the American Educational Research Association, April, Boston, Massachusetts
- Robitaille, D. and K. Travers, (1992), International Studies of Achievement in Mathematics, In D.A., Grouws, Handbook of Research on Mathematics Education, New York: Macmillan Publishing Company, 687-709
- Rotman, B., (2006), Towards a Semiotics of Mathematics, In Hersh, R. 18 Unconventional Essays on the Nature of Mathematics, Chapter 6, 97-127, Springer: New York
- Schoenfeld, A., (1988), When Good Teaching Leads to Bad Results: the disasters of "well-taught" mathematics courses, *Educational Psychologists*, 23(2), 145 – 166
- Shuard, H. and A. Rothery, (1984), Children Reading Mathematics, London: John Murray
- Skemp. R., (1982), Communicating Mathematics: Surface Structures and Deep Structure, *Visible Language*, 281 – 288
- Valverde, G. and L. Bianchi and R. Wolfe and W. Schmidt and R. Houang, (2002), According to the Book: Using TIMSS to Investigate the Translation of Policy into Practice through the World of Textbooks, London: Kluwer Academic Publishers

Free textbooks (aka open textbooks) written by knowledgeable scholars are a relatively new phenomenon. Below, find a meta list of Free Math Textbooks, part of our larger collection 200 Free Textbooks: A Meta Collection. Also see our online collection, 1,500 Free Online Courses from Top Universities.Â Mathematical Methods of Engineering Analysis (PDF) by Erhan Akinlar and Robert J. Vanderbei, Princeton University. Multivariable Calculus by George Cain & James Herod, Georgia Tech.Â The Elements of Statistical Learning by Trevor Hastie, Robert Tibshirani and Jerome Friedman. The Method of Lagrange Multipliers (PDF) by William Trench, Trinity University. The Calculus of Functions of Several Variables by Dan Sloughter, Furman University. This book has been written for the IB Diploma Programme course Mathematics: Analysis and Approaches SL, for first assessment in May 2021. This book is designed to complete the course in conjunction with the Mathematics: Core Topics SL textbook. It is expected that students will start using this book approximately 6-7 months into the two-year course, upon the completion of the Mathematics: Core Topics SL textbook.Â What are your learning and teaching interests in mathematics? I teach mathematics at university but am particularly interested in core pure mathematics which starts in school and continues to be taught at university.Â The textbook will either have comprehensive instructions at the start of the book, specific instructions available from icons located throughout, or both. Textbook serves as an important role in teaching and learning activities. The uses of textbook in the classroom helps teacher in delivering the materials. Similar to the statement above, Mudzakir stated that a textbook is complemented with students work. As he mentioned in his journal, textbook along with many other names is commonly used by educational institution or school and is usually provided with exercise and teaching materials.¹ The standard of students works in a textbook or course book are usually The Process of Textbook Approval: A Critical Analysis. Kahlid Mahmood*. Abstract. Up to the recent past, Textbook Boards were the only institutions in Pakistan that had the mandate to develop textbooks for both government and private schools affiliated with public examination bodies of the country. These boards have been working under their respective provincial governments.Â Approval of the different textbooks in Pakistan is a memorable event. It is an encouraging sign for the quality textbooks.Â This study aims at the critical analysis of the textbooks development process both in government and private sectors, and their approval procedure adopted by the MoECW. The study also analyzes the contents of the approved textbooks.