Five theories of reasoning: Interconnections and applications to mathematics

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Abstract

The last century has seen many disciplines place a greater priority on understanding how people reason in a particular domain, and several illuminating theories of informal logic and argumentation have been developed. Perhaps owing to their diverse backgrounds, there are several connections and overlapping ideas between the theories, which appear to have been overlooked. We focus on Peirce's development of abductive reasoning [39], Toulmin's argumentation layout [52], Lakatos's theory of reasoning in mathematics [23], Pollock's notions of counterexample [44], and argumentation schemes constructed by Walton et al. [54], and explore some connections between, as well as within, the theories. For instance, we investigate Peirce's abduction to deal with surprising situations in mathematics, represent Pollock's examples in terms of Toulmin's layout, discuss connections between Toulmin's layout and Walton's argumentation schemes, and suggest new argumentation schemes to cover the sort of reasoning that Lakatos describes, in which arguments may be accepted as faulty, but revised, rather than being accepted or rejected. We also consider how such theories may apply to reasoning in mathematics: in particular, we aim to build on ideas such as Dove's [13], which help to show ways in which the work of Lakatos fits into the informal reasoning community.

Keywords

informal reasoning; mathematics; Lakatos; argumentation

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It is saying that knowing that the hypothesis of an conditional statement is false allows us to conclude that the conclusion is also false, and we know that this is not valid reasoning. To show that it is not a tautology, we need to find truth assignments for p and q that make the entire proposition false. Since this is possible only if the conclusion if false, we want to let q be true; and since we want the hypothesis to be true, we must also let p be false. It is easy to check that if, indeed, p is false and q is true, then the conditional statement is false. Therefore it is not a tautology. Five theories of reasoning: Interconnections and applications to mathematics. Alison Pease & Andrew Aberdein. Logic and Logical Philosophy 20 (1-2):7-57 (2011). The last century has seen many disciplines place a greater priority on understanding how people reason in a particular domain, and several illuminating theories of informal logic and argumentation have been developed. Perhaps owing to their diverse backgrounds, there are several connections and overlapping ideas between the theories, which appear to have been overlooked. Five theories of reasoning: Interconnections and applications to mathematics. Article. Full-text available. We focus on Peirce's development of abductive reasoning [39], Toulmin's argumentation layout [52], Lakatos's theory of reasoning in mathematics [23], Pollock's notions of counterexample [44], and argumen-tation schemes constructed by Walton et al. [54], and explore some connec-tions between, as well as within, the theories.